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AN APPROXIMATE ANALYTICAL SOLUTION FOR A THREE-DIMENSIONAL HEAT-CONDUCTION PROBLEM IN AN AIR-RADIATION HEATING SYSTEM

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UDC 536.2:536.68

We offer a method for the calculation of the heat transferred from a system, this method being based on replacement of the three-dimensional process by a combination of a two-dimensional and a one-dimensional process in various cross-sectional planes of the heater channels.

The need has recently arisen to conduct thermotechnical calculations related to a variety of design solutions for air-radiation heating systems, governed by the transfer of heat from the surfaces of barriers through whose thicknesses regular channels have been cut, and these are heated by means of circulating hot air (Fig. 1). Rigorous formulation of the steady-state problem of calculating the influx of heat from such a system reduces to the description of the three-dimensional process determined by the Poisson equation, whose precise analytical solution can not be obtained.

We will look for the solution of the formulated problem by taking into consideration the following assumptions: the replacement of the three-dimensional process by a combination of a two-dimensional process within the plane of the lateral cross section of the channels and of the one-dimensional process in the longitudinal cross section of these planes will introduce no significant errors; the temperatures τ_c and t can be assumed to be constant for each lateral cross section of the channel, while the quantities t_1, t_2, α_0, α_1, α_2, λ_0, λ_a, c_a, η_a can be assumed to be constant within the framework of the entire system; we need not take into consideration the heat released from the ends of the barrier, nor need we make provision for the relationship between the amount of heat transferred out of the channel and the location of the latter.

We are familiar with at least three means of solving the two-dimensional heat-conduction problem in the plane of the lateral cross section of regular linear heating elements. Ananikyan's and Pavlov's [1] use of the method of sources and sinks offers no rigorous physical basis and is not applicable to the case $t_1 \neq t_2$.

The solution of the Schwartz–Christoffel integrals (the conformal transformation method) obtained by Sander [2] for the problem in the plane of the lateral cross section of the channels, because of its complexity, leads to resulting differential equations in the longitudinal cross-sectional plane that are insoluble in quadratures.

Most appropriate to the solution of the formulated problem is the Faxen–Rydberg–Huber method. In [3] Faxen published a solution for the two-dimensional heat-conduction problem related to a uniform panel with regular linear heating elements for the case $t_1 = t_2$:

$$\frac{vb}{\pi A}=\frac{k_2-k_1}{k_0}y-|y|+2\frac{\lambda_0}{k_0}+\frac{b}{\pi}\sum_{i=1}^{\infty}\frac{\cos(2\pi ix/b)}{i}\times \quad (1)$$

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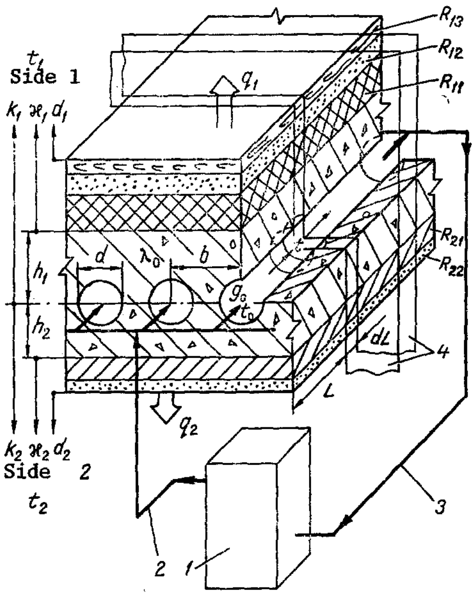

Fig. 1

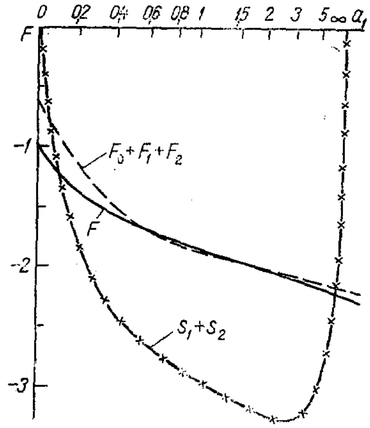

Fig. 2

Fig. 1. Design diagram for the air-radiation heating system of a multilayered barrier: 1) heat generator; 2) entry air channel; 3) reverse air channel; 4) the planes of the theoretical lateral cross sections of the barrier.

Fig. 2. Comparison of the values for the Faxen function (F) with the following approximations: the Rydber-Huber approximation ($S_1 + S_2$); the author's approximation ($F_0 + F_1 + F_2$): $r_1 = 0.01$; $r_2 = 0.01$; $\alpha_2 = 1$.

$$\times\left[\exp\left(-\frac{2\pi i\,|y|}{b}\right)+f_{1i}\exp\left(-\frac{2\pi i y}{b}\right)+f_{2i}\exp\left(\frac{2\pi i y}{b}\right)\right], \tag{1}$$

with the exchange of heat to the panel surfaces being treated as one-dimensional, while the functions contained in (1) were determined from the following equation (here and beyond, the formulas pertaining to side 1 can be symmetrically rewritten for subscripts 1 and 2 for side 2):

$$\nu_0/A = \ln(b/\pi d_э) + 2\pi\lambda_0/bk_0 + F; \tag{2}$$

$$F = \sum_{i=1}^{\infty}(f_{1i}+f_{2i})/i; \tag{3}$$

$$\left(\frac{\alpha_1}{\lambda_0}-\frac{2\pi i}{b}\right)(1+f_{1i})\exp(-4\pi i r_1)+\left(\frac{\alpha_1}{\lambda_0}+\frac{2\pi i}{b}\right)f_{2i}=0, \tag{4}$$

from which the heat released from the surface of the panel can be calculated as follows:

$$\bar{q}_1 = 2\pi A\lambda_0 k_1/bk_0. \tag{5}$$

Rydberg and Huber [4] proposed a generalization of the Faxen method for a multilayered construction in which $t_1 \neq t_2$. The multilayered nature of the structure is taken into consideration by the fact that for the massive middle layer the two-dimensional problem is solved, while for the outer layers it is a one-dimensional problem that is solved, where α is replaced in Eq. (4) by $\varkappa$. However, the inequality of the temperatures t_1 and t_2 is provided for by replacement of ν_0 in Eq. (2) by the conditional quantity derived in the assumption of steady heat conduction of the panel without any heating elements.

In accordance with these assumptions, formulas (2)-(5) can be brought to the following form:

$$\bar{q}_0 = (k_0 t - k_1 t_1 - k_2 t_2)/U; \tag{6}$$

$$\bar{q}_1 = [\bar{q}_0 + k_2(t_2 - t_1)]\,k_1/k_0, \tag{7}$$

where the auxiliary quantity U is defined as

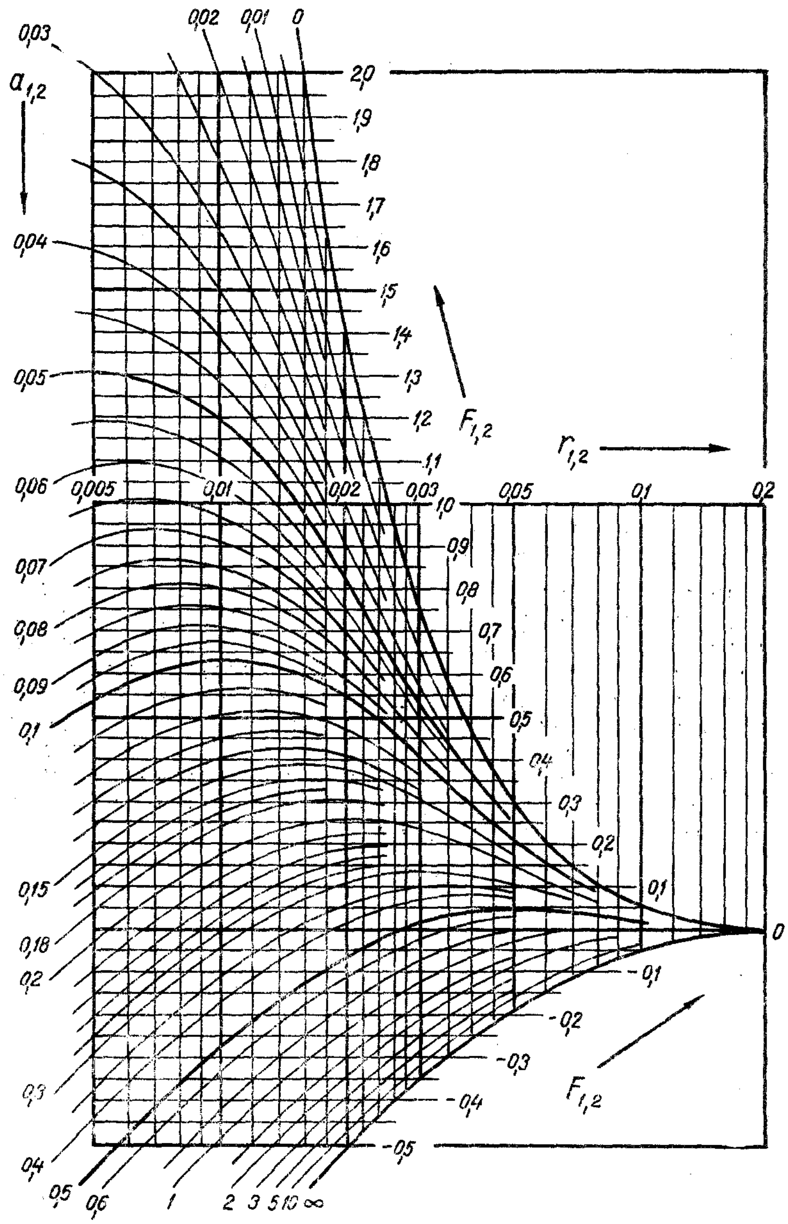

Fig. 3. Dependence of $F_{1,2}$ on $a_{1,2}$ and $r_{1,2}$: $F_{1,2} = -\sum_{i=2}^{\infty} \frac{1}{i(\varphi_{1,2}+1)}$; $\varphi_{1,2} = \frac{a_{1,2} + 2\pi s r_{1,2}}{a_{1,2} - 2\pi s r_{1,2}} \exp 4\pi s r_{1,2}$; $a_{1,2} = h_{1,2}\varkappa_{1,2}/\lambda_0$; $r_{1,2} = h_{1,2}/b$.

$$U = \frac{bk_0}{2\pi\lambda_0}\left(\ln\frac{b}{\pi d_e} + F\right) + 1, \tag{8}$$

while the function F is determined from the following equations:

$$F = \sum_{i=1}^{\infty} \frac{\varphi_{1i} + \varphi_{2i} - 2}{i(1 - \varphi_{1i}\varphi_{2i})}; \tag{9}$$

$$\varphi_{1i} = \frac{a_1 + 2\pi i r_1}{a_1 - 2\pi i r_1} \exp(4\pi i r_1). \tag{10}$$

Since the derived solution is based on the fact that for the massive middle layer it is a two-dimensional problem that is solved, and that it is a one-dimensional problem that is solved for the outer layers, the Biot number for the latter must be less than 0.3:

$$\mathrm{Bi}_1 = \alpha_1 \sum_j R_{1j} < 0.3. \tag{11}$$

However, if owing to the massive nature of any i-th layer inequality (11) is not satisfied, this and the preceding layers must conditionally be connected to the middle layer, with the higher values of h_1 being increased by $\lambda_0 \sum_{j=1}^{i} R_{1j}$.

Let us now solve the one-dimensional problem of heat transfer along the channel. The change in the flow of heat at the inlet to and the outlet from section 4, isolated in Fig. 1, is equal to:

$$dq_0 = g_0 c_a (t_0 - t) - g_0 c_a (t_0 - t + dt) = -g_0 c_a \frac{dt}{dL} \, dL. \tag{12}$$

This flow is passed through the walls of the channel:

$$dq_0 = \bar{q}_0 b dL = b \left(t \frac{k_0}{U} - \frac{t_1 k_1 + t_2 k_2}{U} \right) dL. \tag{13}$$

Having equated (12) and (13), and having solved the differential equation, we find the value of t in the cross section of the channel under consideration here:

$$t = \left(t_0 - \frac{t_1 k_1 + t_2 k_2}{k_0} \right) \exp \left(- \frac{L k_0 b}{g_0 c_a U} \right) + \frac{t_1 k_1 + t_2 k_2}{k_0}, \tag{14}$$

as a result of which we find the sort formula for q_0:

$$q_0 = g_0 c_a \left(t_0 - \frac{t_1 k_1 + t_2 k_2}{k_0} \right) \left[1 - \exp \left(- \frac{L k_0 b}{g_0 c_a U} \right) \right]. \tag{15}$$

Let us now determine the value of q_1. In section 4, isolated in Fig. 1, the following quantity of heat is transferred to side 1:

$$dq_1 = \bar{q}_1 b dL. \tag{16}$$

Having substituted (14) and (7) into (16), and having solved the differential equation, we obtain

$$q_1 = [q_0 + L b k_2 (t_2 - t_1)] \, k_1 / k_0. \tag{17}$$

To determine the values of d_e, found in formula (8), we will use the relationship between the Nusselt and Reynolds numbers for plates with multiple cavities, experimentally derived by Ananikyan [5]:

$$\mathrm{Nu} = 0.038 \mathrm{Re}^{0.72}, \tag{18}$$

from which, at an average air temperature of 60°C in the channel, we have

$$\alpha_0 = \frac{0.038 \lambda_a}{d} \left(\frac{v \rho \, d}{\eta_a} \right)^{0.72} = \frac{3.158}{d} \left(\frac{g_0}{d} \right)^{0.72} \tag{19}$$

(some variations in η_a and λ_a in the selection of some other average temperature exert virtually no effect on the quantities which we are trying to determine). In analogy with [1], having equated the flows of heat from the channel surface and those through some conditional cylindrical layer, we successively obtain:

$$\alpha_0 (t - \tau_c) \pi d = \frac{t - \tau_c}{\frac{1}{2\pi\lambda_0} \ln \frac{d}{d_e}}; \tag{20}$$

$$d_e = d \exp \left[- \frac{\lambda_0}{1.579} \left(\frac{d}{g_0} \right)^{0.72} \right]; \tag{21}$$

$$U = b k_0 \left[\left(\ln \frac{b}{\pi d} + F \right) / 2\pi\lambda_0 + 0{,}1 \left(\frac{d}{g_0} \right)^{0.72} \right] + 1. \tag{22}$$

The practical applicability of these relationships is made difficult by the complexity of calculating the function F which depends on four independent arguments (a_1 , r_1, a_2 , r_2) [see formulas (9) and (10)] and which require rather sophisticated levels of computer technology for their solution. (The Molnar hypothesis [6] to the effect that it is sufficient to calculate F to i = 3 to 4, as a rule, is not uniform. For example, with a and r in the range 0.01 to 0.02 the calculation of F with accuracy to 10^{-2} to 10^{-3} calls for no less than 70-90 steps.) Rydberg and Huber [4] therefore proposed an approximate solution of F that is associated with the separation of the variables φ_{1i} and φ_{2i} and the calculation of the integral exponential function Ei:

$$F \approx S_1 + S_2; \tag{23}$$

$$S_1 = -2\exp(2a_1)\,\mathrm{Ei}(-2a_1 - 2\pi r_1) + \ln[1 - \exp(-4\pi r_1)], \tag{24}$$

which made it possible to construct the relationship S = f(a, r) and to determine F graphically. However, the assumptions made in this case led to significant discrepancy in the quantities ($S_1 + S_2$) and F.

We offer a more precise approximation (see Fig. 2):

$$F \approx F_0 + F_1 + F_2; \tag{25}$$

$$F_0 = \frac{\varphi_{1.1} + \varphi_{2.1} - 2}{1 - \varphi_{1.1}\varphi_{2.1}}; \tag{26}$$

$$F_1 = -\sum_{i=2}^{\infty} \frac{1}{i(\varphi_{1i} + 1)}. \tag{27}$$

This approximation is based on the utilization of the first term in series (9), i.e., F_0 and the sum of the subsequent terms, namely, F_1 and F_2, obtained from series (9) in the assumption that it would be possible to expand this series, i.e., in view of the symmetry of F relative to φ_{1i} and φ_{2i} with $\varphi_{1i} = \varphi_{2i}$. The values of F_1 and F_2 in this case can be determined graphically from Fig. 3.

In final form, the algorithm for the calculation of q_0, q_1 and q_2 concludes with the successive verification of inequalities (11), such as used in Fig. 3, and formulas (10), (26), (25), (22), (15), and (17).

The magnitudes of the channel heat transfer obtained with this algorithm were compared against natural measurements carried out by Ananikyan at the Scientific Research Institute of Health-Physics Engineering (he studied models of series II-04-4 reinforced-concrete plates with multiple cavities streamlined with hot air). This comparison demonstrated excellent convergence of theory and experiment: the average error for the average cavities amounted to 3.8%.

NOTATION

A, auxiliary Faxen function in (1) and (2); $a = h\varkappa/\lambda_0$, structural parameter of the barrier; b, distance between the channels (interval), m; c_a, average specific isobaric heat capacity of air flow, K/(kg·°C); d, channel diameter, m; d_e, equivalent channel diameter, obtained by replacement of internal heat transfer $1/\alpha_0$ by the equivalent resistance of the additional cylindrical layer; Ei(x), integral exponential function; F, basic Faxen function, determined from (3) or (9); $F_{0,1,2}$, approximate functions in (25)-(27); f, additional Faxen functions, determined from (4); g_0, flow rate of air through channel, kg/sec; h, distance from center of channel to outer barrier surface, m; i, number of term in series; $k = 1/(1/\alpha + h/\lambda_0 + \sum_j R_j)$, unilateral heat-transfer coefficient; $k_0 = k_1 + k_2$, total heat-transfer coefficient; L, channel length, m; $Nu = \alpha_0 d/\lambda_a$, Nusselt number; q, heat flow, W, from the outside surface of the barrier of area bL; $q_0 = q_1 + q_2$, total heat transfer from channel, W; $\bar{q}$, density of flow of heat from outer surface in the lateral cross section of the barrier, W/m²; $\bar{q}_0 = \bar{q}_1 + \bar{q}_2$, density of channel heat flow in this cross section, W/m²; R, resistance of the external layer of the barrier to heat transfer, m²·°C/W; $Re = v\rho \cdot d/\eta_a$, Reynolds number; r = h/b, structural parameter of the barrier; S, Rydberg–Huber function, determined from (24); t, t_0, temperature of the air flow, °C in the channel and at the inlet to the channel; $t_{1,2}$, temperature of the outside medium, °C; U, auxiliary function, determined from (8) or (22); $v\rho$, mass velocity of air flow in channel; x, y, coordinates in (1), calculated from the center of the heating element; α, α_0, heat-transfer coefficients, W/(m²·°C) from the outside surface of the barrier and from the inside surface of the channel; η_a, coefficient of dynamic viscosity for the air flow; $\varkappa = 1/(1/\alpha + \sum_j R_j)$, incomplete unilateral coefficient of heat transfer; λ_0, λ_a, coefficients of thermal conductivity for the middle layer of the barrier and for the flow of air; ν, ν_0, excess of temperature at some arbitrary point on the barrier and along the axis of the channel; τ_c, temperature of the channel surface; φ, additional functions determined from (10). Subscripts: i, number of series terms; j, number of outside layer of the barrier, calculated from the middle layer; 1 and 2, external sides of the barrier.

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MODIFICATION OF A FINITE-ELEMENT METHOD TO CALCULATE TEMPERATURE FIELDS AVERAGED OVER ONE COORDINATE

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We examined the approximate solution of an averaged nonsteady boundary-value problem of heat conduction in a two-dimensional region bounded by two continuously differentiable curves.

When we study nonsteady thermal processes, we encounter a need to calculate the temperature field in a two-dimensional region of complex configuration. Difficulties in the solution in the general formulation of the problem lead to a need to develop methods of simplifying the original boundary-value problem. For thermotechnical thin bodies, given a small temperature drop in one of the directions, simplification of the problem is possible by making a transition to temperatures averaged in the appropriate direction. Such a situation arises in the calculation of temperature fields in thin shelves, channels, etc. It is possible, in this case, to simplify the computational procedure involved in studying the dynamics of thermal processes in a region of complex geometry.

The average problem dealt with in this study can be solved by the method of finite elements [1].

Let us examine a two-dimensional region Ω, bounded by two continuously differentiable curves $x_1 = a(y)$, $x_2 = b(y)$, $0 \le y \le d$, $0 < a(y) < b(y)$ (see Fig. 1). We will assume that at the initial instant of time the temperature $\theta_0(x, y)$ of the region is higher than the temperature θ_m of the medium. The transfer of heat from the side surface S of a cylinder, whose cross section is the region Ω, follows the law

$$-\lambda \frac{\partial \theta}{\partial n}\bigg|_S = \alpha \left(\theta\big|_S - \theta_m\right).$$

The original boundary-value problem with boundary conditions of the IIIrd kind has the form:

$$C\gamma \frac{\partial \theta}{\partial \tau} = \lambda \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + f_{sou}(x, y, \tau), \tag{1}$$

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